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IRREGULAR INTERACTION OF WEAK SHOCK WAVES OF DIFFERENT INTENSITIES

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The problem of irregular interaction of weak shock waves, which occurs in the analysis of interpenetration of two waves of different intensities at small interaction angle [1, 2], is considered. It is not possible to solve this problem in linear configuration when the region adjacent to the Mach wave front shrinks to a point, which results in it becoming a nonlinear problem. Behavior of the solution throughout the interaction region is analyzed by the method of matching asymptotic expansions [3, 4]. The external problem is solved in linear formulation. A boundary value problem for the system of nonlinear equations of short waves [5], which takes into account the linking of its solution with the linear external problem and with solutions in the neighborhood of reflected fronts at the inner region boundary, is formulated for the inner region in the neighborhood of the Mach wave front. The effect of the initial state parameters on the pattern of flow is investigated and an approximate solution of the problem is derived.

1. Let us consider the interaction of two plane shock waves in a stationary perfect polytropic gas running off a wedge of angle α (Fig. 1, a). Let the waves meet at instant of time $t = 0$ at point O and begin to interact. We select the system of coordinates so that the Ox-axis lies along the wedge axis of symmetry. For weak shock waves of different intensity

$$P_{\mathrm{I}} = \frac{p_{\mathrm{I}} - p_0}{\varkappa p_0}, \qquad P_{\mathrm{II}} = \frac{p_{\mathrm{II}} - p_0}{\varkappa p_0} \tag{1.1}$$

and small angles α ($\alpha < \alpha^*$) according to [1, 2] an irregular interaction takes place between these, resulting in the formation of a curvilinear front A_1A_2 between the plane

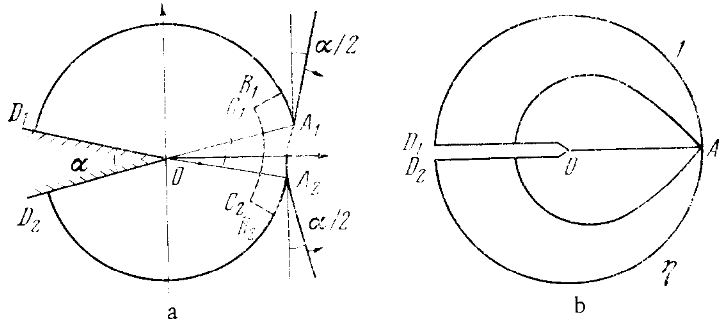

Fig. 1

waves A_1J_1 and A_2J_2 and of a region of perturbation bounded by the wedge surface and the fronts of the Mach and the reflected waves A_1D_1 and A_2D_2 .

Setting $P_{\mathrm{I}} > P_{\mathrm{II}}$ for the parameters of the problem, we have

$$\varepsilon = P_{\mathrm{I}} \ (\varepsilon \ll 1), \ \eta = P_{\mathrm{II}} / P_{\mathrm{I}}, \quad \alpha^{\vee} = \alpha / \sqrt{\frac{\varkappa + 1}{2} P_{\mathrm{I}}} \tag{1.2}$$

In the region of perturbations we have a quasi-stationary (characteristic dimensions of length and time are absent) flow of compressible gas which is vortex-free to within the order of ε^2 and is defined by the equation for the velocity potential [6, 7]

$$\begin{aligned}(1 - r^2) f_{rr} + \frac{1}{r} f_r + \frac{1}{r^2} f_{\theta\theta} = (\varkappa - 1)\left(f - r f_r + \frac{1}{2} f_r^2 + \frac{1}{2r^2} f_\theta^2\right) \times \\ \left(f_{rr} + \frac{1}{r} f_r + \frac{1}{r^2} f_{\theta\theta}\right) + (f_r^2 - 2r f_r) f_{rr} + \frac{2}{r^2} (f_\theta - r f_{r\theta}) f_\theta + \\ \frac{1}{r^4} f_\theta^2 f_{\theta\theta} + \frac{2}{r^2} f_r f_\theta f_{r\theta} - \frac{1}{3} f_r f_\theta^2\end{aligned} \tag{1.3}$$

and that of Lagrange-Cauchy

$$a^{*2} = (1 + \varkappa P)^{(\varkappa-1)/\varkappa} = 1 - (\varkappa - 1)\left(f - r f_r + \frac{1}{2} f_r^2 + \frac{1}{2r^2} f_\theta^2\right) \tag{1.4}$$

$$P = \frac{p - p_0}{\varkappa p_0}$$

In these equations both the independent r, θ and the dependent f, P, a^* variables are related to the components of the Cartesian system x, y, the velocity potential Φ ($u = \Phi_x$, $v = \Phi_y$), pressure p and the speed of sound a by expressions

$$\begin{aligned} x &= a_0 t r \cos\theta, \qquad y = a_0 t r \sin\theta \\ \Phi &= a_0^2 t f(r, \theta), \qquad p = \rho_0 a_0^2 p^*, \qquad a = a_0 a^* \end{aligned} \tag{1.5}$$

where the zero subscript denotes parameters of gas at rest.

Let us establish the conditions at the boundary of the perturbation region. At the wedge walls D_1O and D_2O we have

$$f_\theta = 0 \quad \text{for} \quad \theta = \Pi - \alpha / 2, \quad \theta = \Pi + \alpha / 2 \tag{1.6}$$

At the shock wave fronts $r = k(\theta)$ which in the general case propagate through the uniform stream with potential f_1 at velocity U, the compatibility conditions (in terms of variables (1.5))

$$U - u_n = \frac{2}{\varkappa + 1} a_1^2 (U - u_{1n})^{-1} + \frac{\varkappa - 1}{\varkappa + 1}(U - u_{1n}), \qquad u_\tau = u_{1\tau} \tag{1.7}$$
$$\frac{p - p_1}{\varkappa p_1} = \frac{2}{\varkappa + 1} \left[a_1^{-2} (U - u_{1n})^2 - 1\right], \quad U = k\left[1 + k'^2 k^{-2}\right]^{-1/2}$$

apply. Expressing the normal u_n and tangential u_τ components of velocity in terms of f_r and f_θ, from conditions (1.7) we obtain the differential equation for the shock front

$$f_r = \frac{2k^3 + [(\varkappa + 1) k'^2 + (\varkappa - 1) k^2] f_{1r} + 2k' f_{1\theta}}{(\varkappa + 1)(k^2 + k'^2)} - \frac{2a_1^{*2}}{(\varkappa + 1)(k - f_{1r} + k'k^{-2} f_{1\theta})} \tag{1.8}$$

and the conditions at the front

$$P = \frac{p - p_1}{\varkappa p_1} = \frac{2}{\varkappa + 1} \left[\frac{k^2 (k - f_{1r} + k'k^{-2} f_{1\theta})^2}{a_1^{*2} (k^2 + k'^2)} - 1\right], \quad f = f_1 \tag{1.9}$$

The potential f_1 of the uniform flow upstream of the front is defined by

$$f_1 = \varepsilon \left(b_1 r \cos(\theta \pm \alpha / 2) + b_2\right) \tag{1.10}$$
$$b_1 = q\left(1 + \frac{\varkappa + 1}{2} \varepsilon\right)^{-1/2}, \qquad q = \begin{cases} 1 & \text{on } A_1D_1 \\ \eta & \text{on } A_2D_2 \\ 0 & \text{on } A_2A_2 \end{cases}$$
$$b_2 = -q,$$

where the plus and minus signs relate to the stream ahead of waves A_1D_1 and A_2D_2, respectively.

If the shock wave degenerates into a line of weak discontinuity $r = r_*(\theta)$, then, in accordance with the first of Eqs. (1.7) for $u_n \to u_{1n}$, we have the differential equation of that line

$$r_* \left(1 + r_*'^2 r_*^{-2}\right)^{-1/2} = a_1 + u_{1n} \tag{1.11}$$

Conditions of velocity and pressure continuity

$$f_r = f_{1r}, \quad f_\theta = f_{1\theta}, \quad P = P_1 \tag{1.12}$$

apply along the weak discontinuity line.

Integrating (1.11), we obtain the equation of the weak discontinuity line

$$r = \varepsilon b_1 \cos(\theta \pm \alpha / 2) + \sqrt{a_1^{*2} - \varepsilon^2 b_1^2 \sin^2(\theta \pm \alpha / 2)} \tag{1.13}$$

where coefficient b_1 and the sign of the argument are taken in accordance with (1.10).

The problem thus reduces to the integration of the nonlinear system of Eqs. (1.3), (1.4) with boundary conditions (1.6), (1.8) and (1.9) or (1.12) and (1.13), which presents considerable mathematical difficulties.

2. The solution of the problem of weak shock waves in the perturbation region is usually derived [6] by the method of asymptotic expansion in terms of the small parameter

$$f(r, \theta, \varepsilon) = \varepsilon f^{(1)}(r, \theta) + \ldots, \qquad P = \varepsilon P^{(1)} + \ldots \tag{2.1}$$

whose first terms of expansions $f^{(1)}$ and $P^{(1)}$, obtained from Eqs. (1.3) and (1.4), are defined by the system of linear equations

$$(1 - r^2) f_{rr}^{(1)} + \frac{1}{r} f_r^{(1)} + \frac{1}{r^2} f_{\theta\theta}^{(1)} = 0, \qquad P^{(1)} = r f_r^{(1)} - f^{(1)} \tag{2.2}$$

In linear formulation, by considering the shock wave fronts as weak perturbation fronts and for small angles α transferring the boundary conditions (1.6) at the wedge surface to the axis $\theta = 0$, we eliminate $f^{(1)}$ from system (2.2), use Chaplygin's transformation $\sigma = r^{-1}(1 - \sqrt{1 - r^2})$, and obtain for $P^{(1)}$ the Laplace equation

$$\begin{array}{lll} P_\theta^{(1)} = 0 & \text{for} \quad 0 \leqslant \sigma \leqslant 1, & \theta = \Pi \\ P^{(1)} = 1 & \text{for} \quad \sigma = 1, & 0 < \theta < \Pi \\ P^{(1)} = \eta & \text{for} \quad \sigma = 1, & \Pi < \theta < 2\Pi \end{array}$$

with boundary conditions (1.6) and (1.12).

In conformity with [8] we write the solution of this boundary value problem as

$$P^{(1)} = \frac{1}{2}(1 + \eta) \pm \frac{1}{\pi}(1 - \eta) \operatorname{arctg} \sqrt{\frac{1 - \cos\theta}{\operatorname{ch} \ln \sigma - 1}}, \qquad \eta = \frac{P_{II}}{P_I} \tag{2.3}$$

where the plus and minus signs relate, respectively, to $0 < \theta < \Pi$ and $\Pi < \theta < 2\Pi$. The pressure field in the perturbation region is qualitatively shown in Fig. 1, b in accordance with solution (2.3). Solution (2.3) has a singularity at point A ($r = 1$, $\theta = 0$): pressure changes jump-like from $P^{(1)} = \eta$ along AD_2 to $P^{(1)} = 1$ along AD_1. This singularity is due to the physical imperfection of the linear formulation when the Mach wave front is absent and the pressures along fronts AD_2 and AD_1 are constant. Note that the case of interaction of shock waves of nearly equal intensity ($1 - \eta \leqslant 1$) when the interaction pattern is close to symmertic, cannot be physically considered here as a particular case of solution (2.3) for $\eta \to 1$ and must be analyzed separately.

3. Let us construct the solution defining the flow in the neighborhood of fronts A_1D_1 and A_2D_2. Investigation of each front is conveniently carried out in a moving frame of reference whose velocity coincides with velocity $\bar{q}(q_1, q_2)$ of the uniform flow upstream of the wave front (1.10)

$$\begin{array}{ll} x = a_1 t (x_1 + q_1 / a_1), & y = a_1 t (y_1 + q_2 / a_1) \\ x_1 = R \cos \vartheta, & y_1 = R \sin \vartheta \end{array} \tag{3.1}$$

By expressing the potential of the uniform flow in accordance with (1.10)

$$\begin{array}{l} \Phi_1 = q_1 x + q_2 y + q_3 t \\ q_1 = \varepsilon b_1 a_0 \cos \alpha / 2, \qquad q_2 = -\varepsilon b_1 a_0 \sin(\pm \alpha / 2), \\ q_3 = \varepsilon b_2 a_0^2 \end{array}$$

we obtain a solution of the form

$$\Phi = \Phi_1 + a_1^2 t F(R, \vartheta), \qquad a = a_1 a^{\check{}} \tag{3.2}$$

where a_0 and a_1 are the (dimensional) speeds of sound in the gas at rest and in the uniform stream (1.10), respectively. The relation between components R and ϑ of the moving frame of reference and polar coordinates r and θ is defined in accordance with (1.5) and (3.1).

Passing in the system of equations of gas dynamics to variables (3.1) and (3.2), we obtain for functions F, $P^{\vee}$ and $a^{\vee}$ a system of equations which coincides with system (1.3), (1.4) ($P^{\vee} = (p - p_1) / \varkappa p_1$).

Conditions at the shock front $R = k(\vartheta)$ (1.8), (1.9) are of the form

$$F_R = \frac{2}{\varkappa + 1}\left[\frac{k}{1 + k'^2 k^{-2}} - \frac{1}{k}\right], \quad F = 0, \quad P^{\vee} = kF_R \tag{3.3}$$

Conditions along the line of weak discontinuity whose equation (1.13) in variables R and ϑ is

$$R = 1 \tag{3.4}$$

assume the form

$$F_R = 0, \quad F = 0, \quad P^{\vee} = 0 \tag{3.5}$$

We seek potential F in the neighborhood of the shock wave and of the weak discontinuity line in the form

$$F = \varepsilon^m F_1(\Delta, \vartheta) + \ldots, \qquad R = 1 + \varepsilon^n \varphi(\vartheta)\,\Delta \tag{3.6}$$

In these expressions $\Delta = 1$ corresponds to the shock wave, the equation of the weak discontinuity line is obtained according to (3,4) for $\Delta = 0$, and we can set $\varphi = 1$. Analysis of the behavior of system (1.3), (1.4) at the front (1.8), (1.9) or (1.12), (1,13) shows [4] that $m = 2n$ $(f_{rr} \sim O(1))$. Substituting (3.6) into the equation for F (of the form of (1.3)), for function $Z(\Delta, \vartheta)$ derived with the use of expression

$$F_1 = 2(\varkappa + 1)^{-1}\varphi^2(\vartheta)\, Z(\Delta, \vartheta) \tag{3.7}$$

we obtain the equation

$$2Z_{\Delta\Delta}(Z_\Delta - \Delta) - Z_\Delta = 0 \tag{3.8}$$

similar to the equation of one-dimensional short waves [9]. The general solution of (3.8) is of the form (3.9)

$$Z(\Delta, \vartheta) = {}^1/_2\,|A(\vartheta)|\,\Delta + B(\vartheta) \pm {}^1/_3\sqrt{2\,|A(\vartheta)|}\,[{}^1/_2\,|A(\vartheta)| - \Delta]^{3/2}$$

where ($A(\vartheta)$ and $B(\vartheta)$ are arbitrary functions.

Boundary conditions for $Z(\Delta, \vartheta)$ in accordance with (3.3) and (3.5) are:

at the shock wave front

$$Z = 0, \quad Z_\Delta = 2 \quad \text{for} \quad \Delta = 1 \tag{3.10}$$

at the weak discontinuity line

$$Z = 0, \quad Z_\Delta = 0 \quad \text{for} \quad \Delta = 0 \tag{3.11}$$

From the Lagrange-Cauchy equation (of the form (1.3)) we obtain for pressure at the fronts the expression

$$P = P_1 + \varepsilon^n \varphi^{-1}(\vartheta)\, F_{1\Delta} \tag{3.12}$$

where P_1 defines the pressure upstream of the front in accordance with (1.4); the exponent n and function $\varphi(\vartheta)$ for the shock front, and function $A(\vartheta)$ for the weak discontinuity line are determined by the condition of matching of the internal solution (3.12) with the external solution (2.3) [4].

4. To investigate the flow in the region of shock wave interaction in the proximity of the Mach wave A_1A_2 (Fig. 1, a) we pass to internal variables [4, 7]

$$r = 1 + \frac{\varkappa+1}{2}\varepsilon\delta, \quad \theta = \sqrt{\frac{\varkappa+1}{2}}\varepsilon Y \quad \left(\delta = X + \frac{1}{2}Y^2\right) \tag{4.1}$$

and represent potential f and pressure P in the form

$$f = \varepsilon^2 \frac{\varkappa+1}{2} F^{(1)}(\delta, Y) + \ldots, \quad P = \varepsilon P^{(1)} + \ldots \tag{4.2}$$

Introducing the notation

$$F_\delta^{(1)} = \mu, \quad F_Y^{(1)} = \nu$$

and using Eqs. (1.3) and (1.4), for the first terms of expansion (4.2) we obtain the system of nonlinear equations

$$2(\mu - \delta)\mu_\delta + \nu_Y + \mu = 0, \quad \mu_Y = \nu_\delta, \quad P^{(1)} = \mu \tag{4.3}$$

From (1.8) and (1.9) we obtain the differential equation defining the shock wave front

$$\frac{d\delta}{dY} = \pm\sqrt{2\delta - (\mu + \mu_1)} \tag{4.4}$$

and conditions at the front

$$(\mu - \mu_1)\frac{d\delta}{dY} + \nu - \nu_1 = 0, \quad \mu = P^{(1)} \tag{4.5}$$

Taking into consideration (1.10) for the fronts A_1D_1, A_2D_2 and A_1A_2, we obtain

$$\mu_1 = q, \quad \nu_1 = -q(Y \pm \alpha^\vee / 2) \tag{4.6}$$

The quantity $d\delta / dY$ is geometrically determined by the angle ψ between the normal to the front and the direction of the radius vector [10]

$$\frac{d\delta}{dY} = \pm\psi^\vee = \pm\psi \Big/ \sqrt{\frac{\varkappa+1}{2}P_1} \tag{4.7}$$

In accordance with Eq. (4.3) the condition $\mu = P^{(1)}$ is automatically satisfied at the front. The conditions at the wedge walls (1.6) must be disregarded as external to the region of expansion (4.1).

The system of Eqs. (4.3) represents the known system of equations for short waves [5] and defines the flow in the region of abrupt change of flow parameters downstream of shock waves.

Analysis of condition (4.4) and (4.5) at points A_1 and A_2 of intersection of shock fronts with the condition that in the region downstream of the fronts $\mu = P^{(1)}$ leads to the conclusion [10] that in the first approximation (4.2) the shock fronts A_1D_1 and A_2D_2 are lines of weak discontinuity which in accordance with (4.4) are defined by the equation

$$\delta = \mu_1 \tag{4.8}$$

The derivation of solutions of (4.3) in the internal region requires that in addition to conditions (4.4) and (4.5) the conditions of matching these with the solution in the neighborhood of fronts (3.12) and with the solution (2.3) in the outer region.

Expressing the external solution (2.3) in terms of internal variables (4.1) and retaining the first term of expansions in ε, for the condition of matching (4.3) at the region boundary we obtain

$$\mu = P^{(1)} = \frac{1}{2}[(1+\eta) \pm (1-\eta)] - \frac{1}{\pi}(1-\eta)\operatorname{arctg}\frac{\sqrt{-2\delta}}{Y}, \quad \delta \to -\infty \tag{4.9}$$

where the plus and minus signs relate to $Y \geqslant 0$ and $Y < 0$), respectively.

In the neighborhood of fronts A_1D_1 and A_2D_2 solution (3.12) satisfies conditions (3.11), matches with solution (2.3) for

$$n = 2, \quad B(\vartheta) = -\frac{1}{6}A^2(\vartheta), \quad |A(\vartheta)| = h(\vartheta) = \frac{(\varkappa+1)(1-\eta)}{2\pi \sin\vartheta/2}$$

and for $1-\eta \sim O(1)$ $(h^2 \geqslant 2\Delta)$ is of the form

$$P = \varepsilon\left[\frac{1}{2}(1+\eta) \pm \frac{1}{2}(1-\eta)\right] \pm \varepsilon^2\frac{h^2(\vartheta)}{\varkappa+1}\left(1-\sqrt{1-\frac{2\Delta}{h^2(\vartheta)}}\right) \tag{4.10}$$

where the plus and minus signs relate to $0 < \vartheta < \pi$ and $\pi < \vartheta < 2\pi$, respectively.

The condition of matching the solution of system (4.3) for the first term of expansion (4.2) with solution (4.10) at the region boundary with allowance for (4.1) and (3.6) is

$$\mu = P^{(1)} = {}^1/_2\,[(1+\eta) \pm (1-\eta)], \quad Y \to \pm\infty \tag{4.11}$$

Substituting (4.11) into (4.3) and taking into account the directions of uniform streams (1.10) upstream of weak discontinuity lines, we obtain conditions

$$\nu = -(Y + \alpha^\vee/2), \quad Y \to \infty \tag{4.12}$$

$$\nu = -\eta(Y - \alpha^\vee/2), \quad Y \to -\infty$$

which are equivalent to (4.11).

5. The problem is thus reduced to the integration of the system of Eqs. (4.3) of short waves in region $A_1B_1C_1C_2B_2A_2A_1$ (Fig. 2, a), where the system (4.3) is of the elliptic

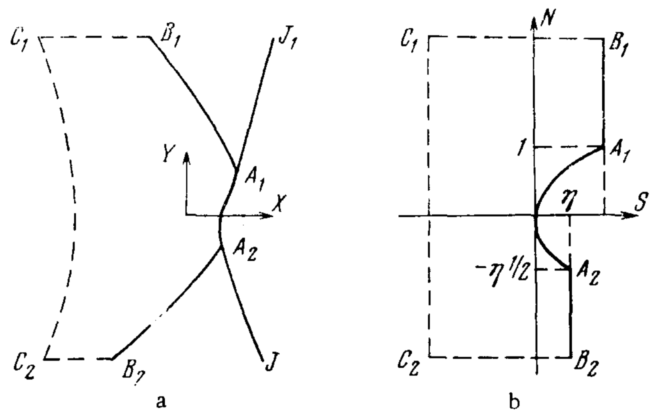

Fig. 2

kind with conditions (4.4), (4.5), (4.8) and (4.12) at the region boundaries

$$
\begin{aligned}
&B_1A_1:\ \mu = 1, \qquad \delta = 1 \\
&A_1A_2:\ \mu \frac{d\delta}{dY} + \nu = 0, \qquad \frac{d\delta}{dY} = \pm \sqrt{2\delta - \mu} \\
&A_2B_2:\ \mu = \eta, \quad \delta = \eta \\
&B_2C_2:\ \nu = -\eta\,(Y - \alpha^{\vee} / 2), \qquad Y \to -\infty, \qquad \delta \leqslant 1 \\
&C_2C_1:\ \mu = \frac{1}{2}[(1+\eta) \pm (1-\eta)] - \frac{1}{\pi}(1-\eta)\operatorname{arctg}\frac{\sqrt{-2\delta}}{Y}, \quad \delta \to -\infty \\
&C_1B_1:\ \nu = -(Y + \alpha^{\vee}/2), \qquad Y \to +\infty, \qquad \delta \leqslant \eta
\end{aligned}
\tag{5.1}
$$

The problem (4.3), (5.1) is a boundary value problem with an unknown element of the region boundary, since the boundary A_1A_2 (the Mach wave front) in (5.1) is determined by the solution $\mu = \mu\,(\delta,\ Y)$, which considerably complicates the analysis and solution of the problem.

Let us pass to new dependent δ and Y and independent S and N variables defined by

$$
S = 2\delta - \mu, \qquad N = -\nu / \mu \tag{5.2}
$$

in which boundary A_1A_2 and the remaining sections of the boundary are known. For the system corresponding to (4.3) in variables (5.2) we have

$$
\begin{aligned}
&2\,(\delta - S)\,Y_N + N\delta_N + (2\delta - S)\,\delta_S - (6\delta - 5S)\,(\delta_S Y_N - \\
&\quad \delta_N Y_S) = 0 \\
&NY_N - \delta_N + (2\delta - S)\,Y_S - 2N\,(\delta_S Y_N - \delta_N Y_S) = 0
\end{aligned}
\tag{5.3}
$$

and boundary conditions

$$
\begin{aligned}
&B_1A_1:\ \delta = 1, \qquad S = 1 \\
&A_1A_2:\ 2N\delta_S + \delta_N - N\,(2NY_S + Y_N) = 0, \qquad N^2 = S \\
&A_2B_2:\ \delta = \eta, \qquad S = \eta \\
&B_2C_2:\ Y = N + \alpha^{\vee} / 2, \qquad N \to -\infty, \qquad S \leqslant 1 \\
&C_2C_1:\ 2\delta + \frac{1-\eta}{\pi}\operatorname{arctg}\frac{\sqrt{-2\delta}}{Y} = S + \frac{1}{2}[(1+\eta) \pm (1-\eta)], \qquad S \to -\infty \\
&C_1B_1:\ Y = N - \alpha^{\vee} / 2, \qquad N \to +\infty, \qquad S \leqslant \eta
\end{aligned}
\tag{5.4}
$$

The problem (5.3), (5.4) is a boundary value problem for the system of nonlinear equations of the elliptic kind ($S < \delta$) in region $G\,(B_1A_1A_2B_2C_2C_1B_1)$ with known boundaries (Fig. 2, b). Transformation (5.2) ensures the one-to-one correspondence of planes $\delta,\ Y$ and $S,\ N$ for inner points of region G, since in accordance with (5.3)

$$
D\,(\delta,\ Y) / D\,(S, N) = \delta_S Y_N - \delta_N Y_S = 0 \quad \text{for} \quad S \geqslant \delta
$$

The formulation of problem (5.3), (5.4) is equivalent to formulation (4.3), (5.1) and is of interest for the mathematical analysis and the exact solution of irregular interaction of two waves of different intensity.

6. Let us investigate the dependence of angle χ between the directions of motion of triple points A_1 and A_2 on initial parameters P_{I}, P_{II} and α or, according to (1.2),

η and $\alpha^\vee$.

Taking into account (5.1), (5.2) and (4.7), we write the expression for N at points A_1 and A_2 of the front A_1A_2 as

$$N_1 = \psi_1^\vee, \quad N_2 = -\psi_2^\vee \tag{6.1}$$

and taking further into account that in accordance with (5.4) $N_1 = 1$ and $N_2 = -\eta^{1/2}$, we obtain

$$\psi_1^\vee = 1, \quad \psi_2^\vee = \eta^{1/2} \tag{6.2}$$

We have the geometric dependence (Fig. 1, a) $\chi + \alpha = \psi_1 + \psi_2$, hence for angle χ we have

$$\chi^\vee = 1 - \alpha^\vee + \eta^{1/2} \tag{6.3}$$

or

$$\chi = \sqrt{\frac{\varkappa+1}{2} p_{\rm I}} + \sqrt{\frac{\varkappa+1}{2} p_{\rm II}} - \alpha \tag{6.4}$$

The considered model of flow obtains for $\chi^\vee \geqslant 0$, i.e. in accordance with (6.3) for $\alpha^\vee \geqslant 1 + \eta^{1/2}$.

Formula (6.4) is an agreement with the experimental data of Smith [1] according to which for $\alpha^\vee \leqslant 1 + \eta^{1/2}$ and weak shock waves the relationship between χ and α is close to linear $(\partial\chi / \partial\theta = 1, \theta = \pi - \alpha)$, while for $\alpha^\vee > 1 + \eta^{1/2}$ χ is very small.

7. Let us consider the approximate solution of problem (4.3), (5.1) using the exact solution of the system of Eqs. (4.3) [5]. We have

$$\begin{aligned}\delta &= -{}^1/_2 Y^2 \mathrm{tg}^2 (b\mu + c) + B \sin^2 (b\mu + c) + \\ &\quad {}^1/_2\ b^{-1} \sin 2 (b\mu + c) + \mu \\ \nu &= [b^{-1} \mathrm{tg}\, (b\mu + c) - \mu]\ Y \quad (b, c, B = \mathrm{const})\end{aligned} \tag{7.1}$$

When $1 - \eta \sim O(1)$, solution (7.1) satisfies conditions (5.1) of matching on C_1C_2 for

$$b = -\frac{\pi}{1-\eta}, \quad \begin{matrix} c^+ = -b & \text{for } Y \geqslant 0 \\ c^- = -\eta b & \text{for } Y \leqslant 0 \end{matrix} \tag{7.2}$$

The conditions along B_1A_1 and A_2B_2 are automatically satisfied in (5.1) because for (7.2) the argument $b\mu + c$ vanishes. The conditions along boundaries C_1B_1 and C_2B_2 are, also, satisfied in (5.1) for $Y \to \pm\infty$ in (7.1).

Using (5.2), (6.1) and (6.2) and taking into account formula (4.6) for ν_1 in the uniform streams downstream of fronts A_1J_1 and A_2J_2, for the coordinates of points A_1 and A_2 we obtain

$$\delta_1 = 1, \ Y_1 = 1 - \alpha^\vee / 2; \ \delta_2 = \eta, \ Y_2 = -\eta^{1/2} + \alpha^\vee / 2 \tag{7.3}$$

At the shock wave front A_1A_2 condition (5.1) (conservation of the tangential velocity component) is approximately satisfied (at points A_1 and A_2 this becomes clear when for $\alpha^\vee \leqslant 2\eta^{1/2}$ (7.3) is substituted into (7.1)). The analysis of solution (7.1) shows that when $Y = 0$ for $\mu = \mu^*$ we have

$$\mu^* = (1 + \eta) / 2 \tag{7.4}$$

We determine constant B in solution (7.1) by integrating the equation of the front of shock wave A_1A_2 (which is conveniently done in variables μ, Y)

$$(d\delta / dY)^2 = 2\delta - \mu$$

from point A_1 to point A_2 with conditions (7.3) so as to have condition (7.4) satisfied for $Y = 0$.

A qualitative picture of pressure distribution in the case of irregular interaction of weak shock waves for $\eta = 0.8$ and $\alpha^{\vee} = 0.2$ is shown in Fig. 3, a. In Fig. 3, b lines

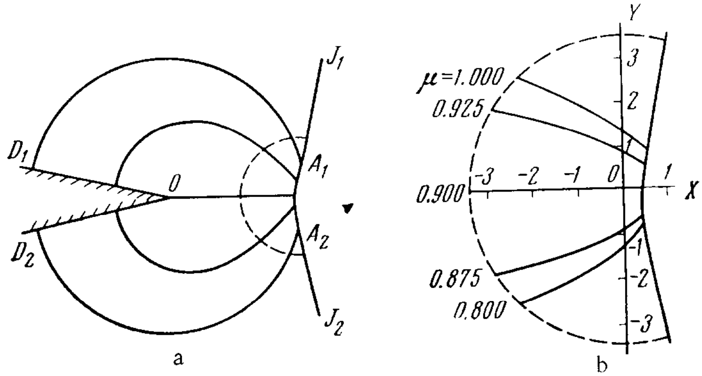

Fig. 3

of constant pressure (velocity) in the neighborhood of front A_1A_2 conforming to solution (7.1) have been plotted in the system of coordinates $X = \delta - {}^1/_2 Y^2$, $Y = Y$ $(B = 0.79)$. The pressure distribution in that neighborhood is consistent with that in the external region where the flow is defined by solution (2.3).

The case of interaction between shock waves of nearly equal intensity $(1 - \eta \ll 1)$ cannot be considered here as the limit for $\eta \to 1$, and must be considered separately. An approximate solution of the problem in the case of irregular reflection of a shock wave from a solid wall, which corresponds to the case of symmetric interaction $(\eta = 1)$ is given in [10].

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PROPAGATION OF ACOUSTIC WAVES IN A MEDIUM LOCATED IN A GRAVITATIONAL FIELD

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Certain problems of acoustic wave propagation in a medium located in a gravitational field are considered on the basis of exact solution for one-dimensional motion of the medium.

1. Fundamental equations and the general solution for one-dimensional motion of medium. Equations of one-dimensional motions of a medium in a gravitational field are of the form

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial x}=-g,\quad \frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+\rho\frac{\partial u}{\partial x}=0 \tag{1.1}$$

This system uniquely defines velocity u and density ρ for a given equation of state $p = p(\rho)$. Introducing new variables w and i, we obtain [1]

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+\frac{\partial i}{\partial x}=0,\quad \frac{\partial i}{\partial t}+u\frac{\partial i}{\partial x}+c^2\frac{\partial w}{\partial x}=0$$

$$\left(w=u+gt,\ di=\frac{dp}{\rho},\ c=\sqrt{\frac{dp}{d\rho}}\right)$$

where c is the speed of sound. After transformation of variables $t = t(w, i)$ and $x = x(w, i)$ the system of equations becomes

$$\frac{\partial x}{\partial i}-u\frac{\partial t}{\partial i}+\frac{\partial t}{\partial w}=0,\quad \frac{\partial x}{\partial w}-u\frac{dt}{dw}+c^2\frac{\partial t}{\partial i}=0 \tag{1.2}$$

Let

$$t=\frac{\partial \psi}{\partial i},\qquad \psi=\psi(w, i) \tag{1.3}$$

then from (1.2) we have

$$x=wt-\frac{\partial \psi}{\partial w}-\frac{gt^2}{2} \tag{1.4}$$

$$c^2\frac{\partial^2 \psi}{\partial i^2}+\frac{\partial \psi}{\partial i}=\frac{\partial^2 \psi}{\partial w^2} \tag{1.5}$$

If the equation of state is given in the form $p = A\rho^n + B$, then $c^2 = i(n-1)$ and the general solution of Eq. (1.5) is